PASSATA and MCAO

Technical Report, version 1.8 (2021/04/13)

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ABSTRACT

The present report describes the algorithms used in PyrAmid Simulator Software for Adaptive opTics Arcetri to model Multi Conjugate Adaptive Optics systems.

1. PASSATA

PyrAmid Simulator Software for Adaptive opTics Arcetri (PASSATA) is an IDL and CUDA based library/software capable of doing Monte-Carlo end-to-end Adaptive Optics simulations. The official reference to it is Agapito *et al.* 2016.[1]

In this document we describe the algorithms used for the implementation of Multi Conjugate Adaptive Optics (MCAO) systems, in particular to simulate MAORY[2] and MAVIS[3] Adaptive Optics (AO) systems.

2. MCAO RECONSTRUCTION AND CONTROL

2.1 Algorithms

2.1.1 Notation

- N_{rl} is the number of reconstructed layers
- N_{ml} is the number of modes of the modal description of the layer or pupil wavefront
- N_{wfs} is the number of wave-front sensors (WFS)
- N_{mw} is the number of measurements (slopes) per WFS
- N_{slp} is the total number of measurements (slopes) = $N_{wfs}\ N_{mw}$
- N_{com} is the length of the commands vector (modal) of all deformable mirrors (DMs)
- N is the number of optimising directions
- $s_{ol}(k)$ are the reconstructed Pseudo Open Loop (POL) slopes computed at loop cycle k (vector of size N_{slp})
- $s_{meas}(k)$ are the measured residual slopes (i.e. WFS measurements) at loop cycle k (vector of size N_{slp})
- s_{ref} are the computed residual reference slopes (reference WFS loop) (vector of size N_{slp})
- C is the atmospheric turbulence covariance matrix of size $N_{rl}N_m \times N_{rl}N_m$, this is a diagonal matrix
- C_N is the WFS noise covariance matrix of size $N_{slp} \times N_{slp}$, this is a diagonal matrix
- D_{DM} is the DM to WFS (slopes) interaction matrix, of size $N_{slp}\ N_{com}$
- D is the reconstructed layers to WFS (slopes) interaction matrix, of size $N_{slp}\ N_{ml}$
- M_i^L is the matrix that does the integration of the atmospheric turbulence in one optimization directions β_i , its size is $N_{ml} \times N_{ml}N_{rl}$

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- M_i^{DM} is the matrix that does the integration of the DMs corrections in one optimization directions β_i , its size is $N_{ml} \times N_{com}$
- w_i is the weight for one optimization direction
- R is the tomographic projector, a matrix of size $N_{rl}N_{ml} \times N_{slp}$
- P is the fitting projector, a matrix of size $N_{com} \times N_{rl}N_{ml}$
- H is the expected noise projector, a matrix of size $N_{com} \times N_{com}$
- $c(k)$ is the DM commands computed at cycle k (vector of size N_{com})
- $\Delta c(k)$ is the error term computed at loop cycle k (vector of size N_{com})
- $a_i,\ b_i,\ i = 1, \ldots, N_{flt}$ are the IIR filter coefficients
- N_{flt} is the order of the IIR filter
- $[]^+$ generalized inverse

2.1.2 Minimum Mean Square Error (MMSE) Estimator

The R matrix that does the tomographic reconstruction of the 3D volume of the turbulence given the measurement can be computed as a Minimum Mean Square Estimator (MMSE):

$$R = \left(D^\top C_n^{-1} D + C_\Phi^{-1}\right)^{-1} D^\top C_n^{-1} \tag{1}$$

or equivalently

$$R = C_\Phi^{-1}\ I^\top \left(DC_\Phi^{-1} D^\top + C_N\right)^{-1} \tag{2}$$

where D is the interaction matrix (dimension: $N_{slp} \times (N_{rl}N_m)$) and C and C_N are diagonal matrices containing the covariance matrices of the layers modal turbulence and the noise of the WFS. R dimensions are $(N_{rl}N_m) \times N_{slp}$.

NOTES:

- C_N is a diagonal matrix, so its inverse is easily computed. C is a block diagonal matrix and its inverse is another block diagonal matrix, composed of the inverse of each block.
- Eq. 1 inverse is "simpler" because $D^\top C_N^{-1} I + C^{-1}$ (dimension $N_{rl}N_m$) is smaller than $DCD^\top + C_N$ (dimension N_{slp}).

2.1.3 Projection from Reconstructed Layers to DMs

The P matrix performs the computation of the phase on a set of optimizing directions (β) from the 3D atmospheric volume computed using R . From these phases, P computes also the optimal correction on the DMs.

In order to build the P we need to write the integration process along all the β directions. Actually, this part corresponds to the concatenations of the M_i^L matrices for each direction. The matrix M^L is then of size $N_{ml}N_\beta \times N_{ml}N_{rl}$, where N_β is the number of optimizing directions (see previous notation description). The matrix multiplication of the M^L for the 3D turbulence vector gives the wave-front in the β directions. Now we need to compute from this the optimal shape of the DMs. Being this a pure geometrical approach, we can build a re-constructor matrix (inverse of an interaction matrix, may be a pure projection matrix) from the phases to the DM modes.

P can be written in the DM modes space (ref. 4) as:

$$P = \left[\frac{\sum_{i=1}^{N_\beta} w_i \left(M_{B_i}^{DM}\right)^\top M_{B_i}^{DM}}{\sum_{i=1}^{N_\beta} w_i}\right]^+ \left[\frac{\sum_{i=1}^{N_\beta} w_i \left(M_{B_i}^{DM}\right)^\top M_{B_i}^{L}}{\sum_{i=1}^{N_\beta} w_i}\right] \tag{3}$$

Where $[]^+$ is the generalized inverse. The dimension of P is $N_{com} \times N_{rl}N_{ml}$.

NOTES:

- Condition number of $\sum_{i=1}^{N_\beta}\left(M_i^{DM}\right)^T M_i^{DM}$ Singular Value Decomposition (SVD) in pseudo (generalized) inversion is generally limited to a maximum value of few hundreds. A Tikhonov regularization proves to be a good choice to solve this generally ill-posed problem. regularization factor used is typically around 1/1000 of the maximum singular value.
- Projection matrix can be used to predict turbulence after a time equal to the delay between measurement and correction, t_d. In fact, if the wind profile, v_{wind}, is known the projection can be done with reconstructed layers moved by $v_{wind}t_d$: this means that M_i^L matrices must be updated accordingly. So, the final DMs shape will account for the turbulence evolution during the delay between measurement and correction. This is valid thanks to the Taylor "frozen turbulence" hypothesis.

2.1.4 Pseudo Open Loop Control (POLC)

The first step is the computation of the Pseudo Open-Loop (POL) measurement from the DMs commands and from the wave-front sensors measurements (ref. 5). In WFS space we write:

$$s_{ol}(k) = s_{meas}(k) + D_{DM}c(k-d)\ , \tag{4}$$

where d should be the total delay in frames to get the exact expression of the pseudo-open loop, where commands and slopes are synchronized. Nevertheless, in the following, we will consider only the case $d = 1$, because, as it can be easily proved, this gives a more robust control than any case with $d > 1$. The error term in DMs space is derived:

$$\Delta c(k) = PRs_{ol}(k) - c(k-1) = PRs_{meas}(k) - Hc(k-1) \tag{5}$$

The matrix product P R has the dimensions of $N_{com} \times N_{slp}$ and $H = I - PRD_{DM}$ has the dimensions $N_{com} \times N_{com}$. Note that this approach is the equivalent to the *implicit POL* presented in Ref. 6.

The matrix H has a clear statistical meaning: it is the expected noise (in the modal space) given the DM commands. We can use the case of equivalence between reconstructed layers and DMs ($P = I$ and $D_{DM} = D$) to show this:

$$\begin{aligned}
H =& I - PRD_{DM} = \\
=& I - RD = \\
=& I - \left(D^\top C_N^{-1} D + C^{-1}\right)^{-1} D^\top C_N^{-1} D = \\
=& \left(D^\top\ C_N^{-1} D + C^{-1}\right)^{-1}\left(D^\top\ C_N^{-1} D + C^{-1}\right) - \left(D^\top\ C_N^{-1} D + C^{-1}\right)^{-1} D^\top C_N^{-1} D = \\
=& \left(D^\top\ C_N^{-1} D + C^{-1}\right)^{-1}\left(D^\top\ C_N^{-1} D + C^{-1} - D^\top C_N^{-1} D\right) = \\
=& \left(D^\top\ C_N^{-1} D + C^{-1}\right)^{-1} C^{-1}
\end{aligned} \tag{6}$$

The reconstructed slopes at each step is given by a combination of the residual phase, the current measurement noise and the measurement noise propagated in the closed loop. R is able to deal with the current measurement noise, while H is used to remove the expected value of the measurement noise propagated in the closed loop and, so, it avoids it propagates further on unseen or not well seen modes. In particular, this is important in MCAO system where a large portion of DMs are not illuminated by Guide Stars (like MAORY[2] and MAVIS[3]).

An Infinite Impulse Response (IIR) time filtering of the error term in DM space is the applied to get the DM commands:

$$c\,(k) = filter_{a_i,b_i}\,(\Delta c(k)) = \frac{1}{a_0}\left(\sum_{i=0}^{N_{flt}} b_i \Delta c(k-i) - \sum_{i=0}^{N_{flt}-1} a_i \Delta c(k-i)\right)\ , \tag{7}$$

being $c\,(k)$ the commands vector for all the DMs in a modal base fixed with the pupil, each DM commands vector needs to be projected into the DM base possibly rotated (post focal DMs) and registered (post focal DMs and M4).

Note that if the filters are pure integrators with scalar gain g, and $\Delta c = PRs_{meas}$, we get:

$$c\,(k) = \left(I - Iz^{-1}\right)^{-1} g\,(\Delta c\,(k) - Hc\,(k-1)) = \left(I - Iz^{-1} + gHz^{-1}\right)^{-1} g\Delta c\,(k) = \left(I - (I-gH)z^{-1}\right)^{-1} g\ \Delta c\,(k)\ . \tag{8}$$

Hence, overall control is a Multi-Input Multi-Output (MIMO) control in the modal space (after $\Delta c = PRs_{meas}$ computation), which joins the dynamic part of the integrator and POL controls. This has a main drawback, because temporal control optimization must be done jointly on integrator and POL control parameters. Naturally, this is true for any IIR time filtering, not only for integrator.

NOTES:

- We can split the SISO and MIMO part of the control as:

$$c\,(k) = \left(I - Iz^{-1}\right)^{-1} g\,(\Delta c\,(k) - H_d c\,(k-1) - H_e c\,(k-1)) = \left(I - (I - gH_d)z^{-1}\right)^{-1} g\,(\Delta c\,(k) - H_e c\,(k-1)) \ , \tag{9}$$

  where H_d is a diagonal matrix with the diagonal elements of H, H_e is a matrix with the extra-diagonal elements of H (that is $H = H_d + H_e$), $\Delta c'(k) = \Delta c\,(k) - H_e c\,(k-1)$ is the MIMO part and $\left(I - (I - gH_d)z^{-1}\right)^{-1} g$ it is a set of SISO leaky integrators with the forgetting factor vector equal to the diagonal of the diagonal matrix $I - gH_d$.
  A rough approximation will be to use only H_d and neglect H_e. This has the advantage of avoiding a MVM in the control.

- We can think of using Eq. 4 to directly control the system as:

$$c(k) = PR\,(s_{meas}(k) + D_{DM}c(k-1)) \ . \tag{10}$$

  Hence, Eq. 8 becomes:

$$c\,(k) = \left(I - PRD_{DM}z^{-1}\right)^{-1} PRs_{meas}(k) \ , \tag{11}$$

  that is equivalent to Eq. 8 with $g = 1$.

2.2 IIR filters and POLC

Note that in general if the filters are IIR ones Eq.8 becomes:

$$c\,(k) = \left(I + \Gamma H z^{-1}\right)^{-1} \Gamma PR\ s_{meas}\,(k) \ , \tag{12}$$

where Γ is a diagonal matrix of filters which j-th diagonal element is:

$$\Gamma_j = \frac{b_0 + b_1 z^{-1} + \cdots + b_{N flt} z^{-N_{flt}}}{1 + a_1 z^{-1} + \cdots + a_{N flt} z^{-N_{flt}}} = \frac{num_{\Gamma_j}}{den_{\Gamma_j}} \ . \tag{13}$$

Then, if we consider only the diagonal of H and $c_{meas} = PR\ s_{meas}$ we can write for the j-th mode:

$$c_j\,(k) = \frac{num_{\Gamma_j}}{den_{\Gamma_j} + num_{\Gamma_j} h_{jj} z^{-1}} c_{meas\,j}\,(k) \ , \tag{14}$$

and:

$$\frac{num_{\Gamma_j}}{den_{\Gamma_j} + num_{\Gamma_j} h_{jj} z^{-1}} = \frac{b_0 + b_1 z^{-1} + \cdots + b_{N flt} z^{-N_{flt}}}{1 + (a_1 + h_{jj} b_0) z^{-1} + \cdots + (a_{N flt} + h_{jj} b_{N_{flt}-1}) z^{-N_{flt}} + h_{jj} b_{N_{flt}} z^{-N_{flt}-1}} \ . \tag{15}$$

3. SPLIT TOMOGRAPHY

Split tomography approach is used to simplify the MCAO management (Ref. 7). Split tomography means that Low Order (LO) – NGS – and High Order (HO) – LGS – have different and independent tomography reconstructors. This is achieved reconstructing Tip, Tilt (on ground layer), Focus and Astigmatisms (on layers with altitude > 0) with NGS measurement in the tomographic volume, while the other modes with LGS measurements. The final reconstruction matrix R is defined as:

$$R = K \begin{bmatrix} R_{LO} & 0 \\ 0 & R_{HO} \end{bmatrix} \tag{16}$$

Where R_{LO} is the LO reconstruction matrix, R_{HO} is the HO reconstruction matrix and K is a matrix that sorts the commands as required by the DMs/reconstructed layers.

Focus and Astigmatisms are reconstructed with NGS measurements because Tip and Tilt are required to solve the ambiguity and understand the correct placement in altitude of these quadratic modes.

Even the matrix the projection matrix P can be split between LO and HO.

NOTES:

- Focus and Astigmatism can be replaced by pure plate scale modes by nulling the total focus and astigmatisms placing opposite focus and astigmatisms modes on ground layer. Hence, LO and HO control are really independent.
- LGS reconstruction matrix must be computed inverting the full LGS interaction matrix, that is with LO modes, otherwise it will not work properly.
- Projection matrix can be splitted or a "full" projection matrix can be used.

4. TRUTH SENSING

The truth sensors are used to measure – slowly varying – LGS correction errors due to truncation effects and non-common path aberrations.

In this work we consider a 3 WFSs, so that truth sensing is able to discern correction errors at different altitudes. Moreover, the offset applied by this loop are modal, and so they will be added to the differential command vector. Hence, Eq. 7 becomes:

$$c(k) = filter_{a,b}\left(\Delta c(k) + r\right) \tag{17}$$

where r is the reference modal offset.

So, using a modal offset instead of a slope offset, one M2V multiplication is avoided. In fact, truth sensing computes a modal vector like the tomographic control and to get slope we need to multiply it by the tomographic control interaction matrix.

5. OPTIONAL FEATURES

5.1 Dual AO

The open-loop commands to the dual AO DM are:

$$u'(k) = \left(P_{NGS}^{turb} - P_{DM}^{NGS}P\right)Rs_{ol}(k) \tag{18}$$

where:

- $u(k)$: unfiltered command to M4 and PF DMs
- $u^{'}(k)$: unfiltered command to the Dual AO DM
- P : Optimization projector (layers DMs)
- P_{NGS}^{turb} : Projection of turbulence to the pupil in the NGS direction (layers pupil)
- P_{NGS}^{DM} : Projection of the DMs phases to the pupil in the NGS direction (DMs pupil)

These commands typically depend on the HO loop only and so the matrix shown in the previous equation are the HO ones.

5.2 Control with different NGS integration times

In this section we consider the case of NGSs with different integration times. This need came from SNR optimization and wind shake rejection. In fact, tip/tilt must reject fast signals induced by wind shake and can benefit from a higher speed.

A first possibility is to use a **forecast** to estimate each LO slope signal at a desired frame rate greater than the one of the NGSs. So, we introduce a forecast function, $F()$, as:

$$\hat{s}_{LO}(k, NGS_i) = F\left(\Delta s_{LO}(j, NGS_i)\right) \tag{19}$$

and the LO commands can be computed as:

$$c_{LO} = R_{TT}\hat{s}_{LO}(k) \tag{20}$$

Instead, below, we describe an approach **without forecast**.

So, we would like to control at different speed tip/tilt and focus/astigmatisms and take advantage of the brightest star. So, let's divide LO reconstruction matrix in two independent parts, tip/tilt and focus/astigmatisms, as:

$$R_{LO} = \begin{bmatrix} R_{TT} \\ R_{FA} \end{bmatrix} \tag{21}$$

Tip/tilt control, c_{TT}, will be driven by the brightest star, NGS_1, to correct for fast signals (coming from wind shake), while the full information of the three stars will be used to get a lower speed tip/tilt offset, c_{refTT} .

$$c_{TT}(k) = filter_{a,b}\left(\Delta c_{TT}(k)\right) \tag{22}$$

$$\Delta c_{TT}(k) = \Delta c_{TT}(k-1, NGS_1)c_{refTT}(k) \tag{23}$$

$$\Delta c_{TT}(k-1, NGS_1) = R_{TT,NGS_1}\Delta s_{LO}(k, NGS_1) \tag{24}$$

$$c_{refTT}(k) = \begin{cases} filter_{a,b}\left(R_{TT}\Delta\bar{s}_{LO}(k)\right) & \text{if } k \text{ mult. } T_1, T_2, T_3 \\ c_{refTT}(k-1) & \text{if } k \text{ not mult. } T_1, T_2, T_3 \end{cases} \tag{25}$$

$$\Delta\bar{s}_{LO}(k) = \sum_{i=k-T_{max}}^{k} \begin{bmatrix} \frac{T_1}{T_{max}}\Delta s_{LO}(i, NGS_1) \\ \frac{T_2}{T_{max}}\Delta s_{LO}(i, NGS_2) \\ \frac{T_3}{T_{max}}\Delta s_{LO}(i, NGS_3) \end{bmatrix} \tag{26}$$

where R_{TT,NGS_1} is the SCAO like tip/tilt reconstruction matrix for the fastest NGS, T_1, T_2, T_3 are the integration times of the three NGS, $T_{\max}$ is the longest integration time.

Focus and Astigmatisms control will go at the slower frame rate, $\max\left([T_1,\ T_2,\ T_3]\right)$.

$$c_{FA}(k) = filter_{a,b}\left(\Delta c_{FA}(k)\right) \tag{27}$$

$$\Delta c_{FA}(k) = R_{FA}\Delta\bar{s}_{LO}(k) \tag{28}$$

Finally:

$$c_{LO}(k) = \begin{bmatrix} c_{TT}(k) \\ c_{FA}(k) \end{bmatrix} \tag{29}$$

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